

# MTSC 887 - Image Processing

## Digital Image Fundamentals

### Project Assignments

Sokratis Makrogiannis, Ph.D.

February 5, 2015

#### **Programming Assignment 1.** Spatial relationships between pixels

Write a program that will find and list the connected components of an image given a starting point, an adjacency type  $N_x$  and a set of intensity values  $V$  to define adjacency.

Use the program to find the connected components of the following arrays using i) 4-adjacency and ii) 8-adjacency with  $V = 1$  starting from the top-left point.

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Write your program in function format, i.e. with input and output arguments. Three of the input arguments should be a numerical array, the starting point coordinates, and the value  $V$ . One of the outputs should be the list of connected components, either as a list of points or as a label/indicator array in which each numerical value corresponds to a connected component.

You can **not** use Matlab's built-in functions for finding connected components.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.

# MTSC 887 - Image Processing

## Digital Image Fundamentals

### Project Assignments

Sokratis Makrogiannis, Ph.D.

January 21, 2015

**Problem 1.** Illumination-reflectance modeling

Assume that a flat area with center at  $(x_0, y_0)$  is illuminated by a light source with intensity distribution

$$i(x, y) = K \cdot e^{-[(x-x_0)^2+(y-y_0)^2]}.$$

Let the reflectance of the area be constant and equal to 4 and let  $K = 127$ . If the digital image is acquired with  $k$  bits of intensity resolution, and the human eye can discern an abrupt change of eight shades of intensity between adjacent pixels, what value of  $k$  will cause visible false contouring?

**Problem 2.** Spatial relationships between pixels

For the below sub-image draw the shortest 4-, 8- and m- path between pixels  $m$  and  $l$  and compute the corresponding lengths. Explain if there does not exist a particular requested path. Solve for i)  $V = \{0, 1\}$  and ii)  $V = \{1, 2\}$ .

$$\begin{bmatrix} (m) & 1 & 1 & 2 & 1 & & \\ & 2 & 2 & 0 & 2 & & \\ & & 1 & 2 & 1 & 1 & \\ & & & 1 & 0 & 2 & 1 & (l) \end{bmatrix}$$

**Problem 3.** Image transformations

Show that the forward and inverse Fourier kernels  $r(x, y, u, v) = e^{-j2\pi(ux/M+vy/N)}$  and  $s(x, y, u, v) = \frac{1}{MN}e^{j2\pi(ux/M+vy/N)}$  are separable and symmetric.

**Programming Assignment 1.** Image interpolation

Download and unzip test images from blackboard page of the course.

Write a program that will

1. read a grayscale image
2. downsample the image by a factor of i) 2 and ii) 8
3. oversample back up to original resolution
4. compute the squared difference between the image of the previous step and the original image
5. display all images and differences
6. compute the average squared difference between the two images.

Repeat the above process for i) nearest neighbor and ii) bilinear interpolation. Show your results on 3 of the grayscale test images.

Write your program in function format, i.e. with input and output arguments. One of the input arguments should be the input image filename so that you can apply your program to any grayscale image. You can use Matlab's built-in functions for resampling.

**Programming Assignment 2.** Image quantization

Download and unzip test images from blackboard page of the course.

Write a program that will

1. read a grayscale image
2. quantize pixel intensities to i) 6 and ii) 3 bits of accuracy.
3. compute the absolute difference between the image of previous step and the original image

4. display all images and differences
5. compute the average absolute difference between the two images.

Show your results on 3 of the grayscale test images.

Write your program in function format, i.e. with input and output arguments. One of the input arguments should be the input image filename so that you can apply your program to any grayscale image. You can **not** use Matlab's built-in functions for quantization.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.

# MTSC 887 - Image Processing

## Intensity Transforms

### Project Assignments

Sokratis Makrogiannis, Ph.D.

February 5, 2015

**Problem 1** (20 points). Histogram equalization

Let a grayscale image with matrix size  $[128 \times 128]$  pixels. Assume that one quarter of image pixels has intensity equal to 8, a second quarter 64, and the remaining half 128. Perform histogram equalization on paper. What are the intensities of the equalized image? Draw graphs of the input and output image histograms.

**Problem 2** (20 points). Intensity transforms

Determine the parameters of exponentials of the form  $e^{-ar^2}$  that implement the transforms of the following figure.

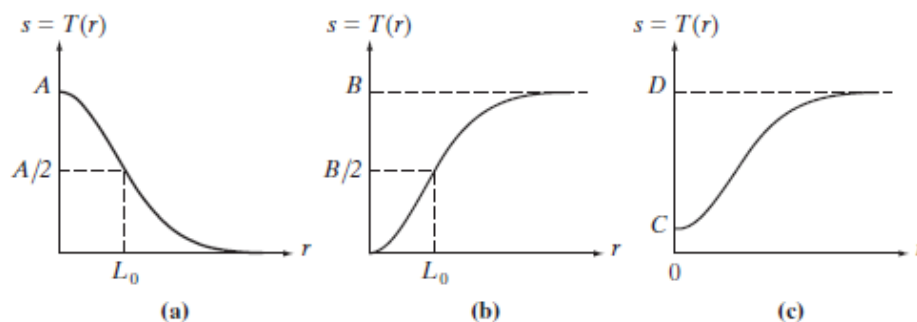


Figure 1: Exponential intensity transforms.

**Programming Assignment 1** (30 points). Histogram calculation

Write a program that will calculate the histogram of pixel intensities of an image using arithmetic operations and no pre-packaged Matlab functions. Write the program in function format, i.e. with input and output arguments. Input arguments should include the input image and the number of histogram bins. The output arguments should include the vector of histogram frequencies. The function should also plot the histogram without using Matlab's "hist" function.

Apply the histogram calculation on two test images from our database, then display the input and output images and the input and output histograms.

**Programming Assignment 2** (30 points). Contrast stretching

Write a program that will perform piece-wise linear transformations of the form  $s = T(r)$  with an arbitrary number of linear segments. Write the program in function format, i.e. with input and output arguments. The input arguments should include the input image, a vector with the  $r_i$ , and a vector with the  $s_i$  coordinates of the linear segments. The output arguments should include the image after contrast stretching.

Apply the histogram calculation on two test images from our database. Define a piecewise-linear transform with i) 3, and ii) 4 segments, display the input and output images, and plot the graph of transformation function.

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# MTSC 887 - Image Processing

## The Fourier Transform

### Project Assignments

Sokratis Makrogiannis, Ph.D.

February 28, 2015

#### **Problem 1.** Fourier Transform Properties

20 points Prove the linearity, duality, and time and frequency shift properties of the one-dimensional continuous Fourier transform. Assuming functions  $f$ ,  $g$ , and their continuous Fourier transforms  $F = \mathcal{F}[f]$  and  $G = \mathcal{F}[g]$ , and constants  $a$  and  $b$ , show the following properties:

- (a)  $a \cdot f + b \cdot g \Leftrightarrow a \cdot F + b \cdot G$
- (b) If  $f \Leftrightarrow F$ , then  $F \Leftrightarrow f$
- (c)  $f(t - t_0) \Leftrightarrow F(\mu) \cdot e^{-j2\pi\mu t_0}$  and  $f(t) \cdot e^{j2\pi\mu_0 t} \Leftrightarrow F(\mu - \mu_0)$ .

20 points Prove both parts of the convolution theorem

- (a)  $f(t) * h(t) \Leftrightarrow H(\mu)F(\mu)$ .
- (b)  $f(t) \cdot h(t) \Leftrightarrow H(\mu) * F(\mu)$ .

**Problem 2** (20 points). Find the Fourier transform of the following functions, with  $t \in \mathbb{R}$

1. 
$$\tau(t) = \begin{cases} 1 - |t| & \text{if } |t| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

You can use the fact that the tent function is the output of convolving a unit rectangular function with itself.



2.  $g(t) = f(t) \cdot \cos(2\pi\mu_0 t)$ , where

$$f(t) = \begin{cases} 4 & \text{if } |t| \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

**Programming Assignment 1** (40 points). Write a program that will implement the forward and inverse discrete Fourier transforms for 1D signals. Write the program in function format, i.e. with input and output arguments. Input arguments should include the original signal and the output arguments should include the transformation output. Do **not** use Matlab's pre-packaged Fourier transform function.

Then create i) a rectangular pulse, ii) an impulse train and ii) a tent function and calculate the respective Fourier transforms. Display the magnitude and phase components and make observations and comparisons with the expected outcomes. Apply inverse transforms to reconstruct signals and compute sums of squared differences to compare reconstructed with original signals.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.

# MTSC 887 - Image Processing Discrete Fourier Transform in 2-D Project Assignments

Sokratis Makrogiannis, Ph.D.

March 22, 2015

**Problem 1** (20 points). Discrete Fourier Transform (DFT) properties.

Let  $f$  be a function in the spatial domain, and  $F$  be a function in the frequency domain, such that  $F(u, v) = \mathcal{F}[f(x, y)]$ , where  $x, y, u, v \in \mathbb{R}$  and  $\mathcal{F}$  denotes the 2-D Discrete Fourier Transform. You can also assume that  $R, I$  are the real and imaginary parts of the function  $F$ , wherever appropriate. Show the following properties of DFT:

1.  $f(-x, -y)$  is complex  $\Leftrightarrow F(-u, -v)$  is complex.
2.  $f(x, y)$  is real  $\Leftrightarrow R(u, v)$  is even and  $I(u, v)$  is odd.

**Problem 2** (20 points). Discrete Fourier Transform (DFT) manual calculation.

1. Calculate the Discrete Fourier Transform of the following signals by hand  
(a)  $\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ , (b)  $\begin{bmatrix} 4 & -3 \\ 2 & 7 \end{bmatrix}$ .
2. Calculate the DFT of the above two cases by hand using the separability property.

**Programming Assignment 1** (30 points). Implementation and testing of DFT in 2-D.

1. Write a Matlab function that implements the forward and inverse Discrete Fourier Transform in 2-D. Write the program in function format, i.e. with input and output arguments. Input arguments should include the original signal and the output arguments should include the transformation output. Do **not** use Matlab's pre-packaged Fourier transform function. Please add comments with description of each step. Try to optimize code for time efficiency.
2. Apply your forward DFT to 2 images from the provided test dataset.
3. Compute and report the root mean square error measure between the frequency magnitude produced by your method and the frequency magnitude produced by Matlab's `fft2` function. Also report and compare the execution times.
4. Display the frequency amplitude and phase components using DC shifting and log transform.
5. Reconstruct images using your inverse DFT.
6. Compute and report the root mean square (rms) error measure between the original image and the reconstructed images to estimate any differences between them.

**Programming Assignment 2** (30 points). Here we will use a property of 2-D DFT to reduce time complexity.

1. Write a function that implements forward and inverse 2-D DFT and using the separability property, i.e. as a sequence of two 1-D transforms.
2. Repeat items 2, 4-6 from above programming assignment, that is i) apply forward 2-D DFT to 2 test images, ii) display frequency amplitude and phase using DC shifting and log transform, iii) reconstruct images using inverse 2-D DFT, and iv) compute and report rms errors between the original and reconstructed images.
3. Use matlab's functions to measure execution time and compare with the previous implementation.

Write the program in function format, i.e. with input and output arguments. Input arguments should include the original signal and the output arguments should include the transformation output. Do **not** use Matlab's pre-packaged Fourier transform function.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.

# MTSC 887 - Image Processing Filtering in Frequency Domain Project Assignments

Sokratis Makrogiannis, Ph.D.

March 30, 2015

**Problem 1** (40 points). Filters in the frequency domain.

1. Given that discrete image intensity derivatives in spatial coordinates  $x$  and  $y$  are calculated by  $\frac{\partial f}{\partial x} = f(x, y) - f(x - 1, y)$  and  $\frac{\partial f}{\partial y} = f(x, y) - f(x, y - 1)$ , i) find the filter transfer function of  $g(x, y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$ , and ii) explain why it behaves as a highpass filter.
2. Given the standard Gaussian filter transfer function, find the Gaussian high pass filter function in the spatial domain for the continuous case.

**Programming Assignment 1** (60 points). Notch and Gaussian band-reject filtering for image restoration.

1. Write a Matlab function that implements i) notch and ii) Gaussian band-reject filtering in the frequency domain. Write the program in function format, i.e. with input and output arguments. Input arguments should include the original image, number of symmetric filters, and the centers and bandwidth for each filter pair. The output arguments should include the filtered image. Please add comments with description of each step.
2. Use the given noisy image to test the efficiency of the above method and compare the performance with a Gaussian band-reject filter. Complete the following steps:
  - (a) Display the frequency amplitude and phase components of the input image using DC shifting and log transform.

- (b) Design a notch band-reject filter to reduce the artifacts of input image and display its filter transfer function in frequency domain.
- (c) Apply filtering using the steps that we discussed in class and lab to reduce wrap-around and ringing artefacts, and other considerations.
- (d) Display filtered image.
- (e) Discuss the above method and its output.
- (f) Repeat steps (b)-(e) for a single Gaussian band-reject filter and compare the results with those produced by the notch filter.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.

# MTSC 887 - Image Processing Image Restoration Project Assignments

Sokratis Makrogiannis, Ph.D.

April 20, 2015

**Problem 1** (40 points). Image degradation.

Consider a linear, position-invariant degradation function  $f$  expressed by  $h(x, y) = e^{-(x^2+y^2)}$ . Suppose that an image is modeled by  $f(x, y) = \delta(y - b)$ , where  $\delta$  is an impulse at  $y = b$ . Assuming no noise, find the expression for the image  $f$  degraded by  $h$  in both the spatial and frequency domains. Use convolution in spatial domain and multiplication in frequency domain to confirm the convolution theorem.

**Programming Assignment 1** (60 points). Image degradation and restoration.

Assume the degradation model  $G(u, v) = H(u, v)F(u, v)$  and the degradation transfer function  $H(u, v) = \frac{T}{\pi(ua+vb)} \sin(\pi(ua + vb))e^{-j\pi(ua+vb)}$ . Write a program that will implement the following steps.

1. Apply motion degradation to a test image from our database using  $a = b = 0.15$  and  $T = 1.1$ .
2. Add Gaussian noise to the degraded image.
3. Restore image using Inverse, Wiener, and Constrained Least Squares filtering. Experiment with different parameter values for each method. Which parameter settings work better and why?

4. Please display the following in your report: i) the degradation function in frequency domain, ii) the frequency amplitude and phase components using DC shifting and log transform for original, degraded and reconstructed images, and iii) original, degraded and reconstructed images in the spatial domain.
5. Compute and report SNR's for each case between restored and original image (before any degradation) to validate the performance of each method.
6. Repeat above experiment for  $a = b = 0.1$  and  $T = 1$ .

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.



# MTSC 887 - Image Processing Mathematical Morphology Project Assignments

Sokratis Makrogiannis, Ph.D.

April 20, 2015

**Problem 1** (40 points). Mathematical morphology theory.

1. Is erosion a commutative operation? Please explain answer.
2. Prove that dilation is commutative and associative.

**Programming Assignment 1** (60 points). Mathematical morphology implementation.

Write a program that will implement morphological erosion, dilation, opening and closing based on the theoretical definitions without using any pre-packaged matlab functions. Use your program to find the boundaries of characters in the text image, using different structuring elements of your choice. Show original and processed images and discuss the results.

**Programming Assignments Write-up** For each programming assignment, you are to turn in a brief report (instructions are posted on blackboard page of the course). The report will determine the grade for each programming assignment. Be well organized, type your reports and include figure captions with a short description of all figures in the report. Motivation and initiative are greatly encouraged and will earn extra points.